A Simplified Correlation Method for Distance Measurement Using Satellites

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To derive range to a moving vehicle for surveillance and navigation purposes, the phase delays in sets of sinusoidal tones may be measured using synchronous satellites for signal relay. Problems of rapid acquisition and measurement occur in multiple-access systems with large numbers of users. This paper describes the selection and processing of an orthogonal tone set which possesses several useful features, particularly the elimination of the requirement for separation of the tones by filtering before measurement. The technique is being used for the Position Location and Aircraft Communications Experiment (PLACE), to be conducted following the launch of the NASA ATS-F satellite. Preliminary experiments have already tested the methods. This paper also describes a simplified, digital technique which performs correlation against the hard-limited signal plus noise and which does not degrade the ranging accuracy. The results of digital simulation are presented.

I. Introduction

THIS paper describes a simple technique for measuring the range to a moving vehicle (e.g., aircraft or marine user) in a proposed navigation system using synchronous satellites as signal relays. In this particular application, which involves the surveillance of a moving vehicle, a measurement is made of the two way range to the vehicle via the satellite relay. The proposed measuring technique derives this range from the phase delays occurring in a set of sinusoidal tones transmitted over the two-way path and received back at the originating station. For distance-measuring accuracy a high tone frequency is required, and successively lower frequency tones are used to resolve the multiple wavelength ambiguities resulting from use of the high frequency tone. It may be shown that for a relay satellite in synchronous orbit, the tone frequency required to resolve the largest possible ambiguity is about 25 Hz. This will allow the determination of the maximum differential between the closest and farthest possible ranges of more than 3191 naut miles.

In one concept of a practical system, the range tones are received continuously by all users but are transponded sequentially in short bursts following a predetermined sequence as shown in Fig. 1 (This shows one satellite of the two required for a position fix). To accommodate a large number of vehicles the length of each tone burst may be less than 0.5 sec, and phase measurements by the ground station must be made for each vehicle in this time.

II. Phase Measurement

In the conventional approach to phase measurement, each received tone is passed through a narrow band filter. This is necessary because the tone is buried in channel noise with a typical S/N ratio of -10 db and this must be raised to 15 or 20 db for successful phase measurement. In addition, filtering is necessary to separate each tone from the others in the set.

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The correlation technique for phase extraction is described in Ref. 1. To determine the unknown phase ϕ of the signal $A\sin(\omega t + \phi)$ relative to a local reference $\sin\omega t$ we generate a quadrature reference $\cos\omega t$ and perform the multiplications:

$$A\sin(\omega t + \phi)\cdot\sin\omega t = (A/2)\cos\phi - (A/2)\cos(2\omega t + \phi) \quad (1)$$

$$A\sin(\omega t + \phi)\cdot\cos\omega t = (A/2)\sin\phi + (A/2)\sin(2\omega t + \phi) \quad (2)$$

Each product is then passed through a low pass filter to remove double-frequency terms and the filter outputs are divided to give

$$\phi = \arctan\frac{(A/2)\sin\phi}{(A/2)\cos\phi} \quad (3)$$

This is shown in Fig. 2. The filtering of the tone is now performed by the low pass filter following the multiplier, but, in general it is still necessary to separate each tone from the others in the set before processing.

The low pass filtering may be carried out by integrating for a time T, and generating

$$I_1 = \int_0^T \frac{A}{2}\cos\phi\cdot dt - \int_0^T \frac{A}{2}\cos(2\omega t + \phi)\cdot dt \quad (4)$$

$$I_2 = \int_0^T \frac{A}{2}\sin\phi\cdot dt + \int_0^T \frac{A}{2}\sin(2\omega t + \phi)\cdot dt \quad (5)$$

The first integrals in Eqs. (4) and (5) go to $[(AT/2)\cdot\cos\phi]$ and $[(AT/2)\cdot\sin\phi]$, respectively, and in-

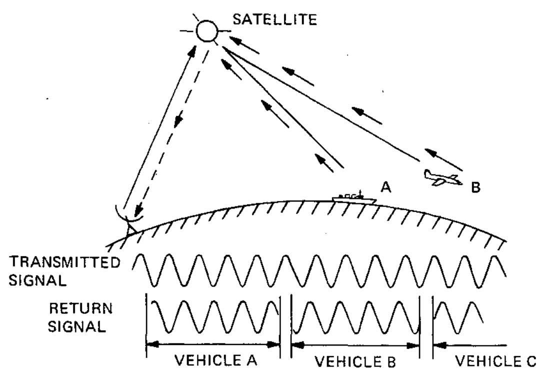

Fig. 1 Sequential tone transponding.

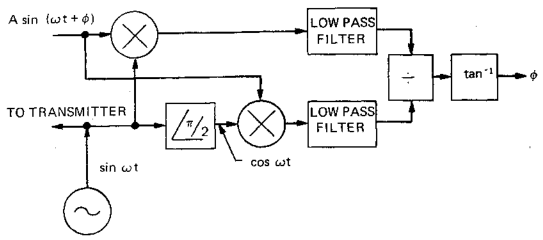

Fig. 2 Signal extraction by correlation.

crease linearly with T while the error term represented by the second integral fluctuates between the values $\pm A$, with increasing T. Thus in general long integration time is required to improve accuracy.

However, it may be seen from Eqs. (4) and (5) that if T is selected such that

$$2\omega T = n \cdot 2\pi \tag{6}$$

where n is a positive integer, then both error integrals will go identically to zero, and the integration time can theoretically be as short as one period ($n = 1$). This approach may be used to measure the phase angle of a desired tone which is a member of a set of tone frequencies, without any previous separation of the tones by filtering. A necessary characteristic of the set is that every component frequency is an integer multiple of some base frequency; i.e., each component can be represented as

$$A_n \sin(m_n \omega t + \phi_n) \tag{7}$$

where A_n is the amplitude, m_n is an integer, ω is the base frequency, and ϕ_n is the arbitrary phase angle of that component. The method follows from the orthogonality of sine waves of frequencies m_n and $m_k (n \neq k)$ over the interval $[0, 2\pi/\omega]$.

A composite set of harmonically related frequencies, $S(t)$ with arbitrary amplitudes and phase angles given by

$$S(t) = A_1 \sin(m_1\omega t + \phi_1) + \ldots + A_k \sin(m_k\omega t + \phi_k) \tag{8}$$

may be resolved into the form

$$S(t) = A_{11} \sin m_1\omega t + A_{12} \cos m_1\omega t + \ldots + A_{k1} \sin m_k\omega t + A_{k2} \cos m_k\omega t \tag{9}$$

where

$$A_j = [(A_{j1}{}^2 + A_{j2}{}^2)]^{1/2} \text{ and } \phi_j = \arctan A_{j2}/A_{j1} \tag{10}$$

The phase of a particular component $A_j \sin(m_j\omega t + \phi_j)$ may be extracted from $S(t)$ by performing the two multiplications and integrations

$$\int_0^{2\pi/\omega} S(t) \cdot \sin m_j\omega t \cdot dt \tag{11}$$

$$\int_0^{2\pi/\omega} S(t) \cdot \cos m_j\omega t \cdot dt \tag{12}$$

Expanding $S(t)$ by Eq. (9) and utilizing the orthogonality relationships yields

$$I_{1j} = A_{j1} \cdot \pi/\omega \tag{13}$$

$$I_{2j} = A_{j2} \cdot \pi/\omega \tag{14}$$

All sum, difference, and double frequency terms integrate identically to zero, and hence

$$\phi_j = \arctan(A_{j2}/A_{j1}) = \arctan(I_{2j}/I_{1j}) \tag{15}$$

It may thus be seen that all the other frequencies have been rejected by the measurement process.

For the scheme under consideration the base frequency ω is 25 Hz. The set $S(t)$ consists of four tones given by

$$\begin{gathered} 25 \text{ Hz};\ 175 \text{ Hz} = 7 \times 25 \\ 1225 \text{ Hz} = 49 \times 25;\ 8575 \text{ Hz} = 343 \times 25 \end{gathered} \tag{16}$$

The minimum integration period to achieve the frequency separation is one of exactly 1/25 Hz or 40 msec. Similarly periods of 80, 120, 160, etc. msec may be used, with the longer integration periods yielding definite advantages for noisy signals. It may be seen how this scheme is ideally suited to the processing of range tones, which are received in short bursts, as shown in the example of Fig. 1. The processing time to be used is the largest multiple of 40 msec which can be fitted into the returned tone burst.

In a practical system there are advantages to using a set of tones which are grouped together at the higher end of the available spectrum. The frequencies to be transmitted in the example under discussion are thus

$$\begin{aligned} 8575 \text{ Hz} &= (343 \times 25) \\ 8575 - 25 = 8550 \text{ Hz} &= (342 \times 25) \\ 8575 - 175 = 8400 \text{ Hz} &= (336 \times 25) \\ 8575 - 1225 = 7350 \text{ Hz} &= (294 \times 25) \end{aligned} \tag{17}$$

Linear combinations of the tones do not destroy their phase ambiguity resolution properties. It may be seen that the use of these simple combinations would present severe problems of signal separation if conventional filtering methods were to be used. The use of the orthogonal filtering technique described above eliminates the necessity of signal separation.

A digital simulation of this scheme was conducted for the noise-free case, the results of which are shown in Table 1. The phase angle of one of the four frequencies of the set was measured for various actual input phase angles. The effect of sampling rate (as a function of the number of samples per cycle of the frequency of interest) was investigated. As shown in Table 1, the effect is insignificant at frequencies above the Nyquist rate.

III. Synchronized Sampling

Use of an actual rate of 4/cycle leads to the first simplification of the practical technique. Figure 3 illustrates sampling at 4/cycle referred to the sine and cosine references used in the correlations. It is seen that by sampling the composite input signal set of four frequencies, $S(t)$, at these instants, the values of sine and cosine to be used for the multiplications to extract one of the components can only be +1, −1, or zero. This allows the multiplication to be performed by simple switching as shown in Fig. 4.

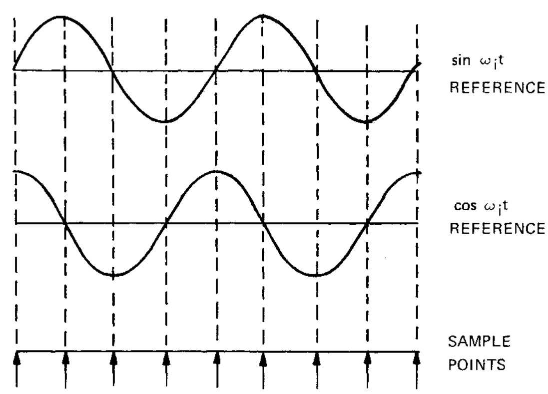

Fig. 3 Sampling at 4 times signal rate.

Table 1 Effect of sampling rate on measurement accuracy

Actual angle, deg	Measured angles, deg Number of samples/cycle 3	10	36
0.0	0.00174	0.00927	0.01283
30.0	30.00948	30.00812	30.02122
60.0	60.01140	60.01303	60.00175
90.0	90.00204	90.00961	90.01311
120.0	120.00974	120.00842	120.02159
150.0	160.01154	150.01317	150.00201
180.0	180.00229	180.00990	180.01340
210.0	210.00980	210.00851	210.00189
240.0	240.01175	240.01334	240.00208
270.0	270.00244	270.01001	270.01323
300.0	300.01001	300.00879	300.02197
330.0	330.01196	330.01343	330.00180
Mean errors	0.00785	0.01407	0.01057

Each analog to digital converter reads the composite signal at a rate synchronized to the frequency of the signal being extracted by that channel. These and the steering switches are slaved to the signal tone generators at the transmitting station. The integrators become accumulating registers and the division and arctangent routines are performed in the computer at the end of the 40 msec processing time.

This technique was digitally simulated, and band-limited gaussian noise with a density of 30 db-Hz was added. The program was set to make 60 measurements of an actual input angle of 180° and the results are shown in Table 2. The theoretical variance for this case is given by the standard formula

$$\overline{\epsilon^2} = \left[\frac{1}{2(P/N_0)\cdot T}\right] \tag{18}$$

where T is the integration time of 40 msec. From this the rms deviation calculates to be 6.6°, which compares with the 6.8° obtained by simulation.

IV. Hard Limiting

A second important simplification may be made when the received composite signal is buried in wide-band noise, as will occur in practice. The A/D converters used at the input to each channel may be replaced by a single hard limiter (output +1 or −1 only). Such an implementation is shown in Fig. 5. The direct or inverted outputs of the limiter are sampled by each channel at its own correct rate and one count is either added or subtracted from the number in the appropriate register. The following analysis is conducted for the special case of the four samples per

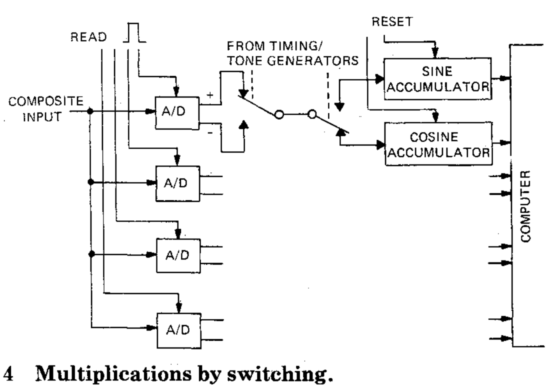

Fig. 4 Multiplications by switching.

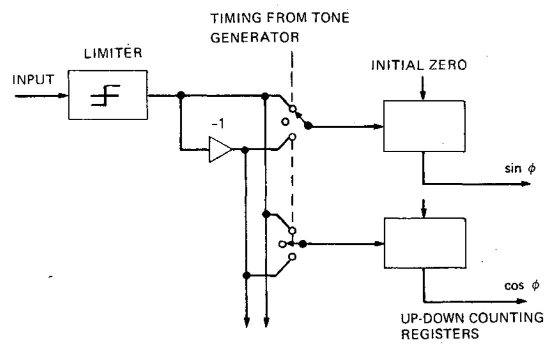

Fig. 5 Digital correlator.

cycle technique, but this is not a requirement for use of the limiter.

Due to the synchronized, four times per cycle sampling, the input signal is sampled at a point where the true magnitude of the desired component is always the same (allowing for a sign reversal as shown in Fig. 6a), since the other two multiplication values are zero. The signal component is assumed to have unit power, [i.e., a peak amplitude of $(2)^{1/2}$] with a total noise of σ^2, where σ is the noise amplitude deviation. The true value of the component at the sample point is $(2)^{1/2}\sin\theta$ where θ is its phase angle, which is to be determined. Similarly the cosine correlation points will occur at true values of $(2)^{1/2}\cos\theta$. Figure 6b shows the criterion that the output of the limiter will have the correct sign. The true voltage V has an additive component of noise (assumed gaussian) of deviation σ. There is a probability P_+ of being above the switching boundary and P_- of being below it, where $P_+ + P_- = 1$. For a large number of samples, n, the number in the register of the correlator will tend to

$$n[(+1)\cdot P_+ + (-1)\cdot P_-] \tag{19}$$

The values of P_+ and P_- may be derived from σ and V (which is a function of θ). To normalize the problem let

$$V = \alpha\sigma \tag{20}$$

then

$$P_+ = 1/2 + \int_0^{\alpha\sigma} p(x)dx \tag{21}$$

$p(x)$ is the probability distribution of the gaussian noise given by

$$\begin{aligned} p(x) &= [1/(2\pi)^{1/2}\sigma]\cdot\exp[-(1/2)x^2/\sigma^2] \\ &= \frac{1}{(2\pi)^{1/2}\sigma}\left[1 - \frac{1}{2}\cdot\frac{x^2}{\sigma^2} + \frac{x^4}{8\sigma^4} - \frac{x^6}{48\sigma^6}\cdots\right] \end{aligned} \tag{22}$$

Taking the first two terms and integrating from 0 to $\alpha\sigma$, yields

$$P_+ = 1/2 + [1/(2\pi)^{1/2}\sigma][\alpha - (1/6)\alpha^3] \tag{23}$$

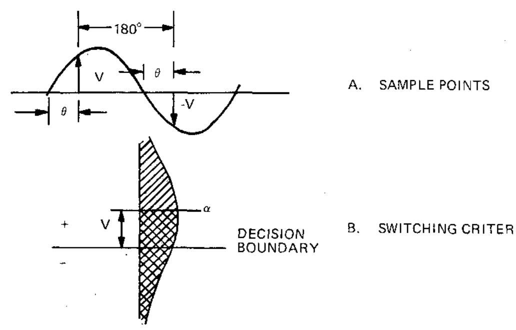

Fig. 6 Sign decision criteria.

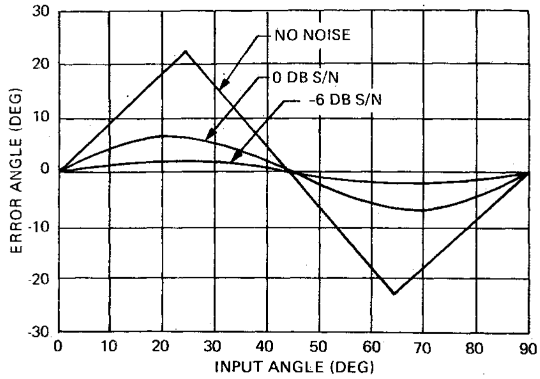

Fig. 7 Hard limited signal bias error.

and

$$P_- = 1/2 - [1/(2\pi)^{1/2}\sigma][\alpha - (1/6)\alpha^3] \quad (24)$$

Since

$$\alpha = V/\sigma = (2)^{1/2} \sin\theta/\sigma \quad (25)$$

the measured value held in the sine correlation register after n samples is given by

$$n \cdot \left(\frac{2}{\pi}\right)^{1/2}\left(\frac{(2)^{1/2}\sin\theta}{\sigma} - \frac{1}{6}\cdot\frac{2(2)^{1/2}\sin^3\theta}{\sigma^3}\right)$$
$$= n \cdot \frac{2\sin\theta}{(\pi)^{1/2}\sigma}\left(1 - \frac{1}{3}\cdot\frac{\sin^2\theta}{\sigma^2}\right) \quad (26)$$

A similar analysis for the cosine register shows that its expected value becomes

$$n \cdot \frac{2\cos\theta}{(\pi)^{1/2}\sigma}\left[1 - \frac{1}{3}\frac{\cos^2\theta}{\sigma^2}\right] \quad (27)$$

The measured angle β is obtained by division and taking arctangent as before, i.e.,

$$\beta = \tan^{-1}\frac{n \cdot \frac{2\sin}{(\pi)^{1/2}\sigma}\left(1 - \frac{1}{3}\frac{\sin^2\theta}{\sigma^2}\right)}{n \cdot \frac{2\cos}{(\pi)^{1/2}\sigma}\left(1 - \frac{1}{3}\frac{\cos^2\theta}{\sigma^2}\right)} =$$
$$\tan^{-1}\frac{\sin\theta\left(1 - \frac{1}{3}\frac{\sin^2\theta}{\sigma^2}\right)}{\cos\theta\left(1 - \frac{1}{3}\frac{\cos^2\theta}{\sigma^2}\right)} \quad (28)$$

It may be seen that as σ gets large (increased noise) then β tends to $\tan^{-1}(\sin\theta/\cos\theta)$ which is the correct result.

The relationship of Eq. (28) is plotted in Fig. 7 to show the error in measurement as a function of actual angle and signal to noise ratio into the limiter. Only the first quadrant of θ is shown since the curve repeats every 90°. The maximum error falls to less than 1° at −6 db S/N and in fact is less than 0.3° at −9 db S/N. Also shown is the "noise free" error, peaking at 22.5°.

Table 2 Sine wave correlation

Noise density:		30 db-Hz
Processing time:		40 msec
Number of runs:		60
True angle	Mean computed angle	rms deviation
180.000°	180.155°	6.829°

Table 3 Limited signal correlation

Noise density:		30 db-Hz
Processing time:		40 msec
Number of runs:		60
True angle	Mean computed angle	rms deviation
180.000°	180.175°	8.222°

A similar analysis for the expected variance of the measured angle leads to the result

$$\overline{\Delta\beta^2} = (\pi\sigma^2/4n)[1 - (2/\pi\sigma^2)\sin^2 2\theta + \ldots] \quad (29)$$

which for large σ tends to

$$\overline{\Delta\beta^2} = \pi\sigma^2/4n \quad (30)$$

where again n is the number of samples taken during the processing time.

The above analysis has considered the measurement of the phase angle of a single tone, buried in wide-band gaussian noise, and has ignored the presence of other tones in the set. For typical S/N ratios of −10 db or less with a small number of tones of equal power (such as the four of this example), the thermal noise power predominates, and the resultant interfering signal tends to a gaussian distribution.

The previous digital simulation was modified to include the proposed "hard limiting" or clipping technique and another series of 60 runs taken, measuring an "actual" fixed 180° angle with a S/N ratio at the limiter of 1/16 or −12 db. This represents a noise density of 30 db-Hz in an 18 kHz channel. Thus σ^2 equals 16 if unity power is assumed in the desired signal. The result of this simulation is shown in Table 3. The value of n is 2 × 343 for the highest tone [from Eq. (17)] since only two of the four samples per cycle are added into each register. Substituting these values into Eq. (30) gives a theoretical rms deviation of 7.76° which may be compared to the simulation value of 8.2°.

V. Conclusions

To extract the phase of each desired tone from a composite range tone set, a correlation technique may be used. If a set of range tones is used in which each component frequency is an integer multiple of a base frequency (specifically 25 Hz), then correlation and integration for a time which is the reciprocal of that frequency (40 msec) provides separation of the frequencies without pre-filtering. Multiples of this processing time may be used to improve the performance in noise, by decreasing the effective bandwidth of the processing filter.

For a sampled-data mechanization, use of a sampling rate synchronized at four times that of the frequency to be measured allows the multiplication functions of the correlation to be replaced by simple digital switching.

Where the received composite signal is buried in wideband noise, the input analog to digital converters necessary for the sampled-data implementation, may be replaced by hard limiters, with a very small degradation of performance.

The feasibility of these techniques has been verified by digital simulation.

VI. Testing

Practical testing has recently been conducted to verify some of the PLACE techniques, by calculating the position of the NASA tracking ship "Vanguard" using range measurements through the two satellites ATS-3 and ATS-5. Analysis of the results of these tests is not yet complete.

Both an analog and a digital implementation of the correlation technique were used. The ATS-3 satellite caused pulse modulation of the transmitted tone set due to its axial spin. An analog correlator was used on the ship as the phase detector in phase locked loops which generated continuous replicas of the range tones, in phase with the intermittently received samples. Accuracies of 0.3° were achieved.

The position of the moving ship was determined in real time by the ground control station at Rosman, N.C., using the phases of the returned ranging tones. The digital phase measuring method with four samples per cycle was employed, using a PDP-11 computer for the division and arctangent routines. Initial data show that phase measurement accuracies of 0.5° are being achieved.

References

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Aircraft Environmental Problems

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The problems facing industry, the airlines, and local communities in a continuing reduction of noise and atmospheric pollution are discussed. Some of the major obstructions currently hindering progress in these areas are defined. Recent research, including ground and flight tests aimed at solving noise problems, is briefly covered. Illustrations are given to demonstrate the importance of accelerating advances in technology and research facilities development, along with suggestions for research that will promote valid and meaningful problem solutions.

Introduction

THE need for relief from irritating aircraft exhaust emission and noise is clearly recognized and progress is being made on a broad front. For example, all the new high bypass ratio engine exhausts are virtually invisible as a result of advances in engine burner technology. In addition, almost the entire fleet of JT8D-powered aircraft, the 727, 737, and DC-9, has been retrofitted with new burner cans in the last two years, such that visible aircraft smoke from commercial airplanes is rapidly becoming insignificant.

Research into means of reducing the impact of aircraft noise is beginning to pay off. All new high bypass engine aircraft are noticeably quieter than the airplanes of a few years back. In addition, we are pleased to be able to say that quiet nacelles for both the 727 and 737 have been developed and are in production. In fact, in the last year and a half, the airlines have purchased 117 of these JT8D-powered aircraft that meet FAR 36 Appendix C noise levels. Sixteen are scheduled for delivery by the end of 1972.

The FAA, NASA, and the airlines have been actively pursuing noise abatement operating procedures that have greatly improved community noise situations in many localities. Avoidance of noise-sensitive areas where possible, high-altitude overflights, steep approaches and takeoffs, reduced or delayed landing flap settings, and power cutbacks are some of the procedures currently being practiced on a routine basis.

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Current Commercial Fleet

In the world today there are over 2840 JT3D and JT8D powered commercial aircraft. Comments are frequently made that these are old aircraft and engines, and should be retired. This is not only completely unrealistic, it is also far from the truth. Both the engines and aircraft have been continuously improved such that those now coming from production lines are generally considered modern in every respect, except for noise and exhaust emissions. Fortunately for the traveling public and those benefiting from shipment by air, the continuous upgrading of these airplanes has resulted in reductions in the costs of air travel and shipping during an extended period of inflation. This ability to keep down costs has been and is of benefit to almost all U.S. citizens, not just to the so-called "jet set."

In addition, the fleet is not really old in terms of accumulated years, even though a few of the airplanes may be approaching retirement age. As seen in Fig. 1, 45% of the JT3D and two-thirds of the JT8D fleets are less than 5 years old. Furthermore, as indicated in Fig. 2, these airplanes are fulfilling an essential role in the air industry. Currently, over three-quarters of the commercial airlines seats are on airplanes powered by JT8D or JT3D engines. There is no way open for these airplanes to be replaced with new aircraft or be retrofitted with new engines in the near future, even if funds were available. The current wide-body jets with new high bypass engines are too large for thin routes that will always be present. No high bypass